Downstream Influence of Film-Cooling in a High-Speed Laminar Boundary Layer

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Nomenclature

B = mass injection parameter
C_f = skin-friction coefficient
H_e = total enthalpy
h = enthalpy
K = ratio of enthalpy
L = characteristic length
M = mass velocity parameter $(\rho v)_w/(\rho u)_\infty$
$\dot{q}_w$ = heat flux at the wall
Re = Reynolds number
St = Stanton number
T = absolute temperature
u = velocity in the stream direction
v = velocity in the y-direction
x = coordinate distance measured from actual origin of boundary-layer growth
x_o = length of porous plate
y = coordinate distance measured perpendicular to surface
β = ratio of enthalpy deficit thickness to momentum thickness
δ = velocity boundary-layer thickness
δ_h = enthalpy deficit thickness
δ_T = thermal boundary-layer thickness
ε_L = compressibility factor
η = film-cooling effectiveness parameter
θ = momentum thickness
ϕ = solid-wall effectiveness parameter
μ = dynamic viscosity
ρ = density

Subscripts

aw = adiabatic wall with coolant injection
c = coolant condition
w = wall (local) conditions
l = local conditions relative to injectant area
o = conditions without injection
∞ = mainstream flow conditions

Theme

A METHOD of predicting the downstream effectiveness in a high speed laminar boundary layer has been developed. The unique feature of the analysis is its simplicity, inasmuch as complex computer codes are not required.

Contents

There have been several pertinent analyses concerning downstream effectiveness resulting from upstream injection in a compressible laminar boundary layer. These include the works of Howe,[1] Rubesin and Inouye[2] and more recently, Wolfram and Walker[3] and Fogaroli.[4]

It has been noted that the above analyses have all required complex computer codes in order to obtain a solution. In the present method, a simple technique is used to determine the downstream effectiveness as a result of upstream injection that gives good engineering accuracy (to within 12% at values of $x/x_o < 1.2$ and to within 4% at values $x/x_o \geqq 2$) without the aid of a computer.

Methodology

The methodology considers a downstream adiabatic plate $(x > x_o)$ with zero injection upstream $(x \leqq x_o)$ and employs the von Kármán momentum equation with the incompressible skin-friction/Reynolds number relations and the Eckert Reference Enthalpy Method, such that

$$Re_\theta = [Re_{\theta_1}{}^2 + 0.44 Re_{\Delta x}\varepsilon_L]^{1/2} \tag{1}$$

where $\varepsilon_L = (\rho^*/\rho_\infty)(\mu^*/\mu_\infty)$ is the compressibility factor (similar to the Chapman-Rubesin parameter) and $Re_{\Delta x} = \rho_\infty u_\infty(x - x_o)/\mu_\infty$. The subscript (1) refers to the heat exchange region (porous) and the superscripts are properties based on reference enthalpy. Moreover, the energy equation, based on enthalpy deficit thickness (δ_h) for an adiabatic wall $(x > x_o)$, can be expressed as

$$\phi = \frac{Re_{h_1}}{Re_h} = \frac{H_e - h_{aw}}{H_e - h_{w_1}} \tag{2}$$

where h_w is the upstream coolant wall gas enthalpy.

If one considers the region $x \leqq x_o$ together with a laminar cubic velocity profile it is easily shown that $\theta/\delta = \delta_h/\delta_T = 0.139$. For $x \gg x_o$, $T \to T_{aw}$ and $\delta_h/\delta_T \to 5/8$. Thus, as $x \to \infty$, $\delta_h/\theta \to 4.5$. On the other hand, if the wall temperature is uniform for $x \leqq x_o$ and for a Prandtl number approximately unity, $\delta_T = \delta$ and $\delta_h = \theta$ but, for $x > x_o$, $\delta_h \neq \theta$. Thus, the temperature profile is confined to the hydrodynamic boundary layer, such that

$$\phi = \delta_{h_1}/\delta_h = \theta_1/\beta\theta \quad \text{for} \quad \beta = \delta_h/\theta$$

and as $x \to x_o$, $\beta \to$ unity; also, as $x \to \infty$, $\beta \to 4.5$.

In the case of a downstream adiabatic impermeable plate with upstream transpiration, an energy balance in the region $0 \leqq x \leqq x_o$ with uniform injectant temperature together with the energy equation gives

$$dRe_h/d(x/x_o) = (\rho v)_w/(\rho u)_\infty Re_{x_o}(1+K) \tag{3}$$

where $K = (h_w - h_c)/(H_e - h_w)$. Integration of Eq. (3) gives $Re_{h_1} = Re_c(1+K_c)$ where $Re_c =$

$$\int_0^{x_o} (\rho v)_w dx/\mu_\infty \quad \text{and} \quad K_c = (h_{w_c} - h_c)/(H_e - h_{w_c})$$

For the adiabatic portion of the plate $(x > x_o)$, it follows from Eq. (2)

$$\phi_{\text{inj}} = \frac{Re_{h_1}}{Re_h} = \frac{1+K_c}{1+K} = \frac{H_e - h_{aw}}{H_e - h_{w_c}} \tag{4}$$

As $x \to \infty$, $Re_{\Delta x} \simeq Re_x \gg Re_{\theta_1}$. Also $Re_\theta \approx Re_h$ along x_o, but $Re_h \simeq \beta Re_\theta$ for $x > x_o$. Consequently the effectiveness η can be expressed as, using Eq. (1),

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$$\eta = \phi_{\mathrm{inj}} = \frac{\beta Re_\theta}{Re_c(K_c + 1)} = \left\{1 + \frac{0.44\beta^2 Re_{\Delta x}\varepsilon_L}{[(1+K_c)Re_c]^2}\right\}^{-1/2} \tag{5}$$

It is normally convenient to consider the injectant temperature as being equal to the porous wall temperature (e.g., $K_c = 0$), whereby, if one considers the limiting case $x \to x_o$, Eq. (5) becomes

$$\eta_{x \to x_o} = \left[1 + \frac{0.44\, Re_{\Delta x}\varepsilon_L}{Re_c^{2}}\right]^{-1/2} \tag{6}$$

In the case of similarity solutions, the mass injection rate is expressed as $(\rho v)_w = Ax^{-1/2}$ and the blowing parameter at the wall as

$$f(0) = -2\frac{(\rho v)_w}{(\rho u)_\infty}\left[\frac{\rho_\infty u_\infty x}{\varepsilon_L \mu_\infty}\right]^{1/2} = -B \tag{7}$$

for B constant. From Eqs. (5) and (7) we can write

$$\eta_{x \to x_o} = \left[1 + \frac{0.44}{B^2}\left(\frac{x}{x_o} - 1\right)\right]^{1/2} \tag{8}$$

For a uniform mass injection upstream, one obtains

$$(\rho v)_w = B(\rho u)_\infty\left[\frac{\rho_\infty u_\infty x_o}{\varepsilon_L \mu_\infty}\right]^{1/2} \tag{9}$$

and combining the above with Eq. (5) results in the same expression as given by Eq. (8). Thus, Eq. (8) is valid for both the similarity injection and uniform injection cases.

Discussion and Conclusions

In Fig. 1 the film-cooling effectiveness is presented as a function of the dimensionless distance ratio x/x_o. The dotted curves in this figure represent the finite-difference solutions of Fogaroli[4] for uniform and similarity upstream injection, respectively. It is observed that the difference in injection distribution is slight, and that this difference is uniform with distance. Moreover, the present technique is shown to agree quite favorably with the more exact computer solution. It should be noted that the limiting case of $x \to x_o$ ($\beta \to$ unity) was considered inasmuch as the dimensionless distance ratio (x/x_o) presented in Ref. 4 was limited to a value of four.

Thus, it has been demonstrated that the incompressible phenomenological laws governing laminar boundary-layer flow, when modified for compressibility effects, can be used to predict film-cooling effectiveness for high-speed flow. Moreover, a superposition of heat-transfer rates can be employed to predict heat transfer effects over a solid wall downstream of an upstream injection region. The technique is quite general and is capable of employing any distribution of upstream injection to suit engineering design requirements.

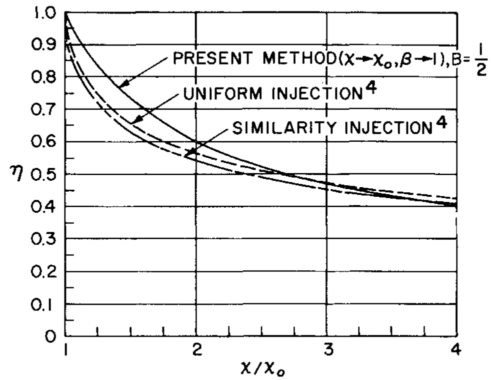

Fig. 1 Downstream effectiveness with upstream injection.

References

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